A novel approach to identifying hedonic demand parameters

Nicolai V. Kuminoff\textsuperscript{a,}\textsuperscript{*}, Jaren C. Pope\textsuperscript{b,1}

\textsuperscript{a} Department of Economics, Arizona State University, Main Campus PO Box 873806, Tempe, AZ 85287-3806, United States
\textsuperscript{b} Department of Economics, Brigham Young University, 180 Faculty Office Building, Provo, UT 84604, United States

\section{Introduction}

Rosen’s (1974) first-stage model of hedonic pricing is among the foremost tools of empirical microeconomics. It is routinely used to estimate the value of small changes in the characteristics of workers, private goods, public goods, and externalities.\textsuperscript{2} In comparison, Rosen’s vision for a second-stage model of demand remains unfulfilled. The problem is identification. There are two general strategies for identifying demand parameters. One is to make explicit assumptions about the structure of preferences in a single geographic market (e.g. Bajari and Kahn, 2005). The other is to collect data from multiple geographic markets, assuming that consumers in each market share a common preference structure (e.g. Palmquist, 1984, Bartik, 1987, Zabel and Kiel, 2000). Neither approach has been widely applied.

The purpose of this note is to present a new solution to Rosen’s second stage. We demonstrate that unexpected changes in the composition of a differentiated product can be used to identify demand parameters from data on a single geographic market. Our work provides a new perspective on studies that have sought to use quasi-experiments to improve hedonic modeling. In these studies, researchers have nested first-stage hedonic models within panel data frameworks that aim to estimate marginal values from microdata on prices and characteristics measured before and after shocks to product quality, information, and institutions (e.g. Card, 1990, Card and Krueger, 1992, Hirsch, 1993, Davis, 2004 and Pope, 2008). When these shocks are not marginal, they have the potential to generate instruments that can solve the identification problem with second-stage demand estimation. This is our main point.

Section 2 presents an equilibrium model of a differentiated product market. Section 3 uses the model to explain how the instruments are generated. Section 4 concludes.

\section{Hedonic equilibrium}

Price-taking consumers purchase a single unit of a good differentiated by \( k \) characteristics, \( x = [x_1, \ldots, x_k] \). The utility
maximizing problem is
\[
\max_{x, b} U(x, b; \alpha) \quad \text{subject to } y = b + P(x; \theta).
\]  
(1)

Each consumer chooses characteristics and the numeraire composite commodity \(b\) to maximize utility, given her preferences \((\alpha, \gamma)\), income \((y)\), and the price schedule \(P(x; \theta)\), which depends on a parameter vector, \(\theta\). The first-order conditions are
\[
\frac{\partial P(x; \theta)}{\partial x} = \frac{\partial U}{\partial b} = D(x; \alpha, y).
\]  
(2)
The consumer chooses a good that provides levels of each characteristic at which her marginal willingness to pay (MWTP) for an additional unit equals its marginal implicit price. Assuming that marginal utility of income is constant, the second equality in (2) observes that as \(x\) varies, the marginal rates of substitution define inverse demand curves.

Let \(C(m, x; \beta)\) denote a producer’s cost function, where \(m\) is the number of type-\(x\) goods that they sell and \(\beta\) is a vector of parameters differentiating producers. In addition to reflecting heterogeneity in production technology, \(\beta\) also reflects heterogeneity in exogenous factors affecting the characteristics of the good supplied. Examples of exogenous factors used in conjunction with hedonic models include unexpected changes in health risk (Davis, 2004), weather (Aschenfelter, 2008), desegregation (Card and Krueger, 1992), and information disclosure laws (Pope, 2008).

Price-taking producers are free to vary the number of units they sell as well as the characteristics of each unit. The profit maximization problem is
\[
\max_{x, m} \pi = m \cdot P(x; \theta) - C(m, x; \beta),
\]  
(3)

with corresponding first-order conditions
\[
P(X; \theta) = \frac{\partial C(m, x; \beta)}{\partial M},
\]
\[
\frac{\partial P(x; \theta)}{\partial x} = \left(\frac{1}{m}\right) \frac{\partial C(m, x; \beta)}{\partial x}.
\]  
(4)

Producers choose \(m\) to set the offer price of the marginal unit equal to its production costs, and they choose \(x\) to set the marginal cost of each characteristic equal to its implicit price.

Equilibrium occurs when the first-order conditions are simultaneously satisfied for all consumers and producers. This system of differential equations implicitly defines the equilibrium hedonic price function that clears the market (Rosen, 1974). It will be useful to rewrite the price function to highlight its dependence on model primitives,
\[
P(x; \theta) = P[x(A, B); \theta(A, B)].
\]  
(5)
Equilibrium prices and quantities are determined by all of the exogenous variables: \(A: F(y, \alpha) \sim A\), a vector of parameters that describes the joint distribution of income and preferences and \(B: V(\beta) \sim B\), a parameter vector describing the distribution of producer characteristics.

Fig. 1 provides a stylized picture of how the equilibrium price function reveals the distribution of marginal values for each characteristic. It relates the marginal price function for \(x_1\) to demand curves for two consumers and supply curves for two producers. Evaluating \(\partial P(\cdot)/\partial x_1\) at a consumer’s chosen level of \(x_1\) will return their MWTP. Combining this information with \(x_1\) identifies exactly one point on their demand curves.

3 Tinbergen (1959) developed this linear–quadratic–normal model in order to consider the properties of equilibria in labor markets with heterogeneous workers. Epple (1987) and Eckeland et al. (2004) use the model to illustrate other features of hedonic equilibria.
foreach consumer type, tracing out two points on their demand curves. An unexpected shock to a product characteristic changes MWTP of consumer preferences function of the structural parameters describing the distributions for characteristic of a differentiated product, using data from a simple example illustrates how the sorting process that underlies marginal price function to shift. The shift traces out two points further improvement in price of the good but decreases each consumer’s MWTP for a demand is downward sloping, a positive shock increases the shock, as well as demand curves for two consumers. Because demand is downward sloping, a positive shock increases the price of the good but decreases each consumer’s MWTP for a further improvement in \( x_1 \). In aggregate, these decreases cause the marginal price function to shift. The shift traces out two points on the demand curve for each consumer “type” in the figure. This simple example illustrates how the sorting process that underlies adjustment between different equilibria can identify the demand for characteristics of a differentiated product, using data from a single geographic market.

Our identification strategy requires repeated cross-section data describing consumers and their choices, straddling a market shock. ⁴ Econometric analysis would proceed in two stages: (i) estimate single-period price functions before and after the shock, and then (ii) use the shock to develop instruments for demand estimation. The first stage exploits the nonlinearity of the marginal price function (Ekeland et al., 2004). The second stage exploits the mechanics of the hedonic model to recover demand curves.

4. Conclusion

We have suggested a novel approach to hedonic demand estimation. The identification derives from the ways in which heterogeneous consumers adjust their behavior to unexpected changes in the composition of the differentiated product. Their collective adjustments can produce shifts and rotations in the gradient of the price function that effectively trace out market demand curves. Limiting the analysis to a single geographic market enhances the validity of the estimates by improving the comparability of the “before” and “after” groups (Meyer, 1995).

Rosen (2002) first called our attention to the fact that changes in the gradient of the price function provide additional information about preferences and technology, beyond what is revealed by a single equilibrium. He suggested using this information to adjust price indices for advances in technology that decrease the cost of living. We have outlined a counterpart to his proposal. Changes in the hedonic gradient may also serve to identify the demand for characteristics of goods and services that improve the quality of life.

References


⁴ Data on repeated cross-sections of consumers are now widely available from commercial vendors in the form of scanner data and real estate assessment databases.